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STIMULATED ELECTRON COOLING

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Abstract Electron cooling in a uniform magnetic field is calculated by using a simple quantum mechanical model. The possibility of the drastic reduction of the cooling time is consequently discussed when an electron beam is irradiated with the electromagnetic field, of which frequency is close to the cyclotron frequency of the electron.

INTRODUCTION

It is well known that the electron cooling rate of heavy charged particles (ions) is greatly accelerated in a longitudinal magnetic field[1] when

1) the longitudinal temperature of a electron beam is negligible, and

2) the transversal velocities of ions are much smaller than those of electrons.

This is understood by the fact that an ion collides with the Larmor rings, not with point electrons. This means that the effective relative velocity is reduced from $| \vec{v}_p - \vec{v}_{e\perp} | \simeq \vec{v}_{e\perp}$ to $\vec{u} = \vec{v}_p - \vec{v}_{e\|}$ and results in the enhancement factor of $< \vec{v}_{e\perp} >^3 / < \vec{u} >^3$.

The idea of stimulated electron cooling is based on the quantum mechanical consideration of electron states in a uniform magnetic field. These states are known as Landau orbits. The energy level of each orbit is characterized by ω_c (cyclotron frequency) and corresponds to the transversal energy of an electron. When the electromagnetic wave with the frequency near ω_c is irradiated, electrons wander very actively among these orbits, then ions may see each electron as a Larmor disk rather than a Larmor ring. This would results in an enhancement in the collision rate. Contrary to the classical treatment where the transversal motion of electron does not take any part in the energy exchange because this motion is always fixed by the external magnetic field, there is a significant contribution to the energy exchange through the excitations among Landau orbits by the ion- electron collision. This collision with orbit excitations is stimulated by the external radiation without any limitations mentioned in the beginning of this section.

In this paper, the following facts are assumed for the simplicity.

a) An ion is a proton.

b) The longitudinal temperature of the electron beam is negligible.

c) The interaction between protons and electrons is considered in the electron rest flame.

COULOMB SCATTERING IN A UNIFORM MAGNETIC FIELD

The motion of an electron in a uniform magnetic field is described by the wave function [2,3]

$$| m,s >= \phi_{m,s}(\rho,\theta)e^{i(k_m z-\omega_m t)} \quad ,$$

$$\phi_{m,s}(\rho,\theta) = \sqrt{\tfrac{\gamma_c}{\pi}}e^{i(m-s)\theta}\sqrt{\tfrac{s!}{m!}}d^{(m-s)/2}e^{-d/2}L_s^{m-s}(d)$$

Here the direction of the magnetic field is taken as the z-axis in the cylindrical coordinates, $d = \gamma_c\rho^2, \gamma_c = \frac{1}{2}m_e\omega_c/\hbar, m_e$ is the electron mass, s and u are non-negative integers, and $L_s^\ell(d)$ is a Laguerre polynomial. The energy of the electron is $\hbar\omega_m$ where $\omega_m = (m+\frac{1}{2})\omega_c + \frac{1}{2}\hbar k_m^2/m_e$.

The wave function of an incident proton is approximated as

$$| p_i >= (2\pi)^{-3/2}e^{i(\vec{k}_i\vec{r}_i-\omega_i t)} \quad , \quad \omega_i = \tfrac{1}{2}\hbar\vec{k}_i^2/m_p \quad , \quad m_p\text{: proton mass}$$

The S-matrix element for the scattering with exciting the m-th Landau orbit to the n-th one and given angular momenta of an electron is given in the first Born approximation by

$$S = -(i/\hbar)\int_{-\infty}^{+\infty} < n,u : p_f \mid V \mid m,n : p_i > dt \quad ,$$

$$V = \frac{e^2}{|\vec{r}_p-\vec{r}_e|}e^{-a|\vec{r}_p-\vec{r}_e|} = \frac{e^2}{2\pi^2}\int\frac{e^{i\vec{K}(\vec{r}_p-\vec{r}_e)}}{K^2+a^2}d^3\vec{K} \quad .$$

With the careful consideration of the integral of Laguerre polynomials,[4] the scattering cross section is given by

$$\frac{d\sigma(m,s:n,u)}{d\varphi dk_n} = \frac{4\alpha^2m_p^2c^2}{\hbar^2k_i}\frac{|g(m,s:n,u)|^2}{K_1^4} \quad , \tag{1}$$

$$g(m,s:n,u) = (-1)^{s+u}(-i)^\nu\sqrt{\tfrac{n!s!}{m!u!}}t^{\frac{\nu}{2}}e^{-t}L_s^{u-s}(t)L_n^{m-n}(t) \quad , \tag{2}$$

where $\vec{K}_1^2 = \vec{K}^2 + a^2, \vec{K} = \vec{k}_f - \vec{k}_i, K_z = k_m - k_n, t = \vec{K}_\perp^2/(4\gamma_c), \nu = m-n-s+u$, α is the fine structure constant, and φ is the angle between the x-axis and the transversal momentum of the scattered proton. The cross section with the transition from the m-th orbit to the n-th one is obtained by summing over the final angular momenta, u, and averaging the initial ones, s. In this calculation, the s-distribution of the initial electron beam is supposed as $\sim (s!)^{-1}$. Consequently this cross section is given by

$$\frac{d\sigma(m,n)}{d\varphi dkn} = \frac{1}{e}\sum_{s,u=0}^{\infty}\frac{1}{s!}\frac{d\sigma(m,s:n,u)}{d\varphi dkn} \quad .$$

According to the assumption b), k_m is always set zero in the followings.

MAGNETIZED COOLING

Longitudinal Cooling

In this case, $\vec{k_i} = k_i\hat{z}$ and $K_1^2 = 4\gamma_c t + k_n^2 + a^2$ where $4\gamma_c t = k_n(2k_i - rk_n) - 2m_p \cdot (n-m)\omega_c/\hbar$ and $r = m_p/m_e$. The energy gain of the recoil electron, $T_o(m,n)$, due to the scattering from the m-th orbit to the n-th one, is calculated from the equations 1) and 2) for $m = 0 \sim 2, n-m = 0 \sim 3$, and $0 \le s,u \le 40$. The results are shown in Fig. 1. The scattering with $n - m = 1$ is dominant and their energy dependence is about $T_p^{-0.63}$ where T_p is the incident proton energy.

From Eq. (2) and the expression of K_1^2, it is clear that the contribution from small t is dominant. Then the approximated estimate is obtained as, for example,

$$T_o(m \neq 0, n = m+\ell) \simeq \sigma_t^0(m,n)\ell\hbar\omega_c \simeq \frac{4\pi\alpha^2\hbar^2c^2r}{T_p}F_{m,\ell}\Delta t \quad ,$$

$$F_{m,\ell} = \frac{(m+\ell)!\,(\ell+1)^{\ell-1}\ell\ e^{-\frac{\ell+1}{2m}}}{2^{2\ell+1}m!\ m^{\ell-2}(\ell!)^2(m+B^{-1})^2} \quad , \quad B = \frac{1}{\ell+1}\left(\ell^2\frac{r\hbar\omega_c}{T_p} + \frac{a^2}{\gamma_c}\right) \quad .$$

Numerically this approximation gives

$$T_o(m, m+\ell)(eV \cdot cm^2) = 4.8 \times 10^{-10}T_p^{-1}(eV)F_{m,\ell}\Delta t \quad .$$

Because the main purpose of this paper is not to calculate exactly, but just to estimate the order of magnitude, the region of the integral in this approximation, Δt, is so taken as to give the same T_p-dependence as in the exact numerical calculation, i.e. $\Delta t \simeq 0.5\ T_p^{0.37}(eV)$.

The total energy loss of the proton per collision is then obtained as

$$< T_p >_o = \sum_{m,\ell} T_o(m, m+\ell)\rho_m, \quad \rho_m = b\ e^{-bm}, \quad b = \hbar\omega_c/T_e \quad ,$$

where ρ_m and T_e are the transversal energy distribution and the transversal temperature of the electron beam. Using the approximations,

$$\sum_{\ell=1}^{\infty} T_o(0,\ell) \simeq 1.7\ T_o(m \neq 0, m+1) \quad ,$$

$$\sum_{\ell=1}^{\infty} T_o(m \neq 0, m+\ell) \simeq 1.4\ T_o(m \neq 0, m+1) \quad ,$$

we get

$$< T_p >_0 \simeq 3.1 \sum_{m=1}^{\infty} T_o(m, m+1)\rho_m \quad \simeq 3.1 \times \tfrac{1}{2}\pi\alpha^2\hbar^2c^2rT_p^{-1}\Delta t$$
$$\simeq 5.1 \times 10^{-3} v_{||}^{-1.26}(cm/sec)\ (eV \cdot cm^2) \quad . \tag{3}$$

Figure 2 shows the experimental results at INP [5] and the curve of Eq. (3). This approximation well agree with the data. In these approximate calculations, the expression of $F_{m,\ell}$ is adjusted to agree with the one obtained from the asymptotic form of a Laguerre polynomial when m >> 1.

Transversal Cooling

The direction of the incident proton beam is taken along the x- axis as $\vec{k}_i = k_i\hat{x}$. Suppose the scattering angle of a proton is very small, we can modify Eq. (1) as

$$d\sigma(m, s : n, u) \simeq \frac{4\alpha^2 m_p^2 c^2}{\hbar^2} \frac{|g|^2}{K_1^4}(1 - r \cdot cos^2\theta/2)d\varphi d(cos\theta),$$

where θ is the angle between the scattered proton and the z-axis, therefore $\varphi \simeq 0$ and $\theta \simeq \frac{\pi}{2}$. Then taking only the terms of $n - m = 1$, we get the normalized cooling time as

$$\tau_N = n_e\tau = \frac{m_p v}{2<T_p>_0} \simeq \frac{m_e\gamma_c^{1/2}}{\alpha^2\hbar c^2\Delta_0} v_\perp^2$$

$$\simeq 10^{-12} f_c^{1/2}(GHz)\Delta_0^{-1} v_\perp^2(cm/sec) \quad (sec \cdot cm^{-3}) \; .$$

When $f_c \simeq 2$ GHz, the cone angle of the scattered proton beam, Δ_0, should be 10^{-4} to make this approximation consistent with the INP data [5] shown in Fig. 3. Anyway we can derive the velocity dependence of the cooling time.

STIMULATED ELECTRON COOLING

Now we introduce a radiation field. The electron state is perturbed as,

$$| \, e >= e^{i(k_m z - \omega_m t)}\{| \, m, s > +A_{m+1} \, | \, m+1, s > e^{-i\omega t}$$
$$+A_{m-1} \, | \, m-1, s > e^{i\omega t}\} \; , \tag{4}$$

$$A_{m\pm 1} = -i\sqrt{m + \tfrac{1}{2} \pm \tfrac{1}{2}} C \; e^{\mp i\chi}/\Delta\omega \quad ,$$

$$C = eE\gamma_c^{-1/2}\hbar^{-1}/4 \quad , \qquad \Delta\omega = \omega_c - \omega + i\Gamma/2 \quad ,$$

where dipole interaction between the electron and the electric field is assumed, E and ω are the amplitude and the frequency of the electric field, $\omega \simeq \omega_c$, m/Γ is the life of the m-th Landau orbit, and χ is the relative phase between E and the transversal motion of the electron. Then Eq.(2) is modified in the first order by

$$g'(m:n) = g_0(m:n) + g_A(m:n) + g_E(m:n),$$

$$g_A(m:n) = A_{m+1} g_0(m+1:n) + A^*_{n-1} g_0(m:n-1),$$

$$g_E(m:n) = A_{m-1} g_0(m-1:n) + A^*_{n+1} g_0(m:n+1),$$

where the suffixes s and u are omitted and $g_0(m:n)$ is given by Eq. (2). Supposing $g_0(m:n+1)$ is dominant and neglecting the additive contributions of the interference terms, we get

$$\sum_n | g'(m:n) |^2 = | g_0(m:m+1) |^2 \qquad (n = m+1)$$
$$+ | A_{m+1} |^2 \{ | g_0(m:m+1) |^2 + | g_0(m+1:m+2) |^2 \} \quad (n = m+2)$$
$$+ | A_{m-1} |^2 | g_0(m-1:m) |^2 + | A_{m+1} |^2 | g_0(m:m+1) |^2 \qquad (n = m) \,.$$

This gives the energy loss of the proton due to the scattering by the electron dressed with photons in the m-th Landau orbit as

$$T_m = T_m^0 + 2 \, | A_{m+1} |^2 \, (T_m^0 + T_{m+1}^0),$$

where $T_m^0 = T_0(m, m+1)$. The enhacement factor of the energy loss of the proton per collision is given by

$$F = < T_p > / < T_p >_o \simeq 1 + 4 (\tfrac{C}{\Delta\omega})^2 \tfrac{T_e}{\hbar\omega_c}$$

$$= 1 + \frac{2\alpha T_e I(\omega)}{\hbar m_e \omega_c^4 \delta^2} \simeq 1.6 \times 10^8 \; T_e(eV) I(\omega)(W/cm^2) f_c^{-4}(GHz) \; \delta^{-2} \quad , \qquad (5)$$

where $\delta = 1 - \omega/\omega_c$, T_e is the transversal temperature of the electron beam, and $I(\omega)$ is the power density of the radiation.

Even though $| A_{m\pm1} | < 1$ for the application of Eq. (4), the enhancement factor would reach $\sim T_e/(\hbar\omega_c)$. In case of $| A_{m\pm1} | > 1$, the electron state is not correctly described by Eq. (4), but we can expect that Eq. (5) gives the first order or the minimum factor. Many problems, for example, the effect of the non-negligible k_m or how to confine the radiation in the electron cooling region, remain to be solved in future.

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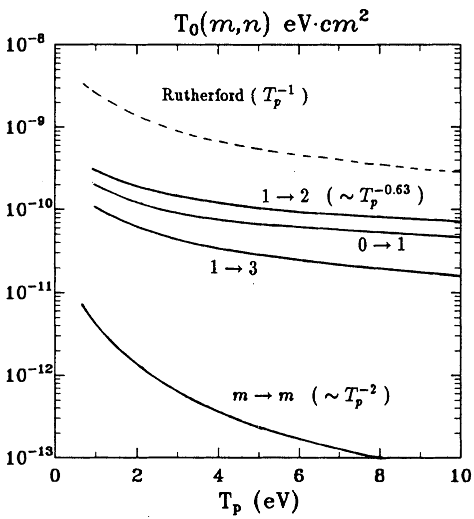

FIGURE 1 Energy gain of a recoil electron per collision.

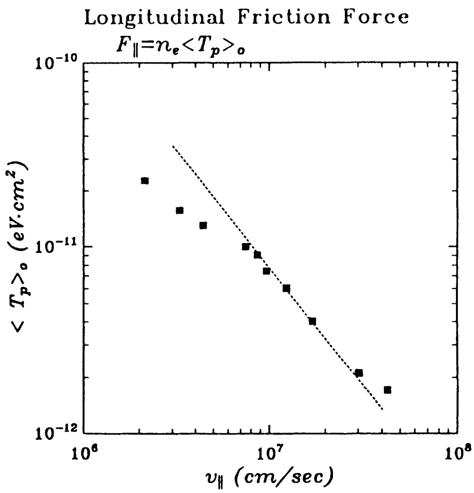

FIGURE 2 Longitudinal friction force.

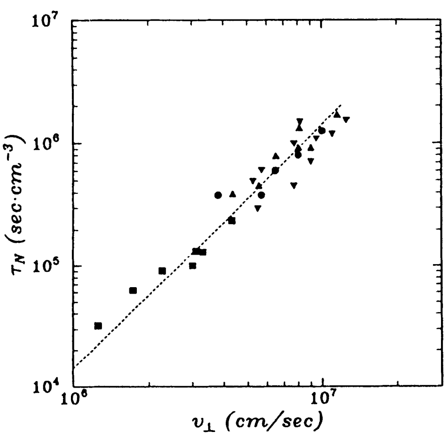

FIGURE 3 Normalized transversal cooling time.